Statistics

Fall 2022

Lecture 9

Continuous random variable with prob. dist.

SG 18 – SG 21

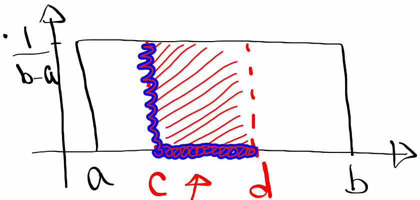
- Uniform Prob. dist.
- Standard normal Prob. dist.
- Normal Prob. dist.
- Central-limit theorem
- Applications.

Exam 2 is in 2 weeks.

Uniform Prob. Dist.

Let x be a continuous random variable for all values from a to b with Uniform Prob. dist.

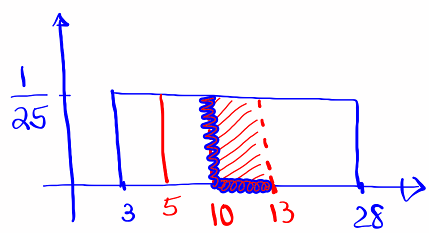
1) Graph is rectangular from a to b with width of $\frac{1}{b-a}$



2) $P(x=c)=0$

3) $\boxed{P(c<x<d)}$ is the area within the rectangular shape. $=(d-c)\cdot\frac{1}{b-a}$

Consider a Uniform Prob. dist for all values from 3 to 28.

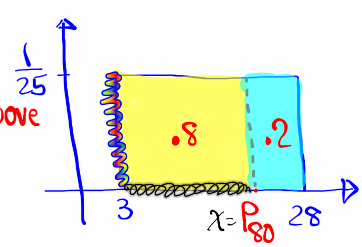


1) $P(x=5)=0$

2) $P(10<x<13)=(13-10)\cdot\frac{1}{25}=\boxed{\frac{3}{25}}=\boxed{.12}$

3) Find $x=P_{80}$

80% below   20% above

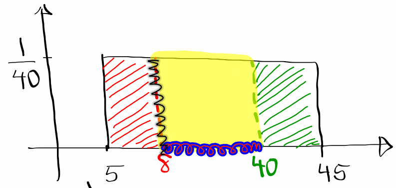


$(x-3)\cdot\frac{1}{25}=.8$

$x-3=25(.8)$

$x-3=20$ $\rightarrow$ $\boxed{x=23}$

Consider a Uniform Prob. dist. for all values from 5 to 45.



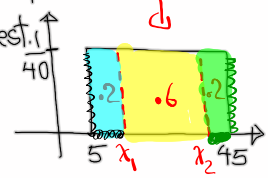
1) Find $P(x<8 \text{ OR } x>40)$

$= 1 - P(8<x<40) = 1 - (40-8)\cdot\frac{1}{40}$

Total Prob. 1

$= 1 - \frac{32}{40} = 1 - \frac{4}{5} = \boxed{\frac{3}{5}}$

2) Find two x-values that separate the middle 60% from the rest.



$1 - .6 = .4$

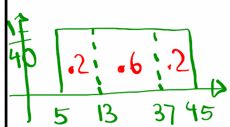
$.4 \div 2 = .2$

$(x_1 - 5)\cdot\frac{1}{40} = .2$

$x_1 - 5 = 40(.2)$

$x_1 - 5 = 8$

$\boxed{x_1 = 13}$



$\boxed{x_2 = 37}$

$(45 - x_2)\cdot\frac{1}{40} = .2$

$45 - x_2 = 40(.2)$

$45 - x_2 = 8$

$45 - 8 = x_2$

Standard Normal Prob. dist.

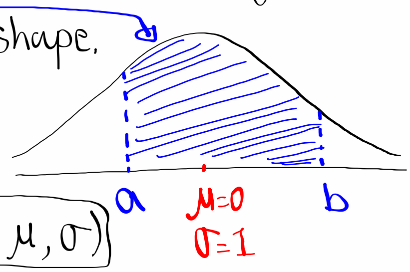
1) Use Z, $P(Z=c) = 0$

2) Dist. is symmetric, bell-shape, with total area $=1$.

3) Mean = Mode = Median

4) $\mu = 0$, $\sigma = 1$

$\boxed{P(a<Z<b)}$ is the corresponding area within the bell-shape.



How to find it:

$\boxed{\text{2nd}}$ $\boxed{\text{VARS}}$

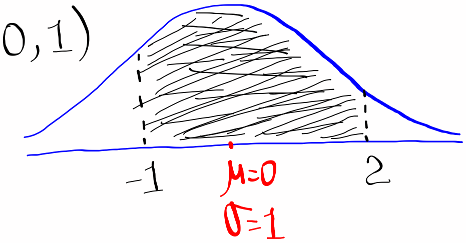
$\boxed{\text{normalcdf}(L, U, \mu, \sigma)}$

Find $P(-1 < Z < 2)$

$= \text{normalcdf}(-1, 2, 0, 1)$

$(-)$

$= \boxed{.819}$



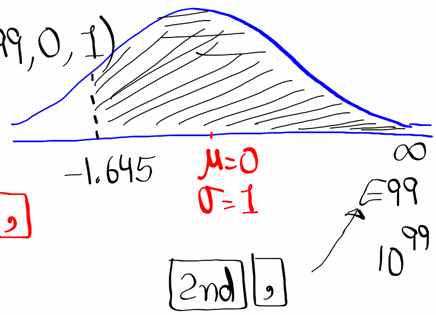
Find $P(Z > -1.645)$

$= \text{normalcdf}(-1.645, E99, 0, 1)$

$(-)$

$\boxed{2nd}\ \boxed{,}$

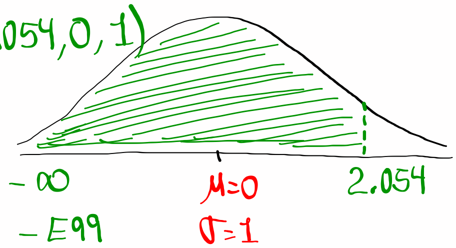
$= \boxed{.950}$



Find $P(Z < 2.054)$

$= \text{normalcdf}(-E99, 2.054, 0, 1)$

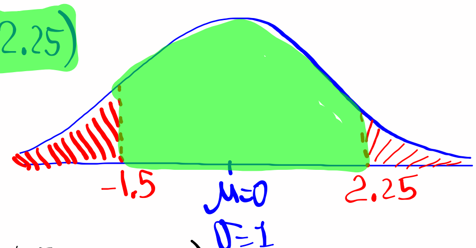
$= \boxed{.980}$



Find $P(Z < -1.5 \text{ OR } Z > 2.25)$

$= 1 - P(-1.5 < Z < 2.25)$

↑
Total
Prob. 1



$= 1 - \text{normalcdf}(-1.5, 2.25, 0, 1)$

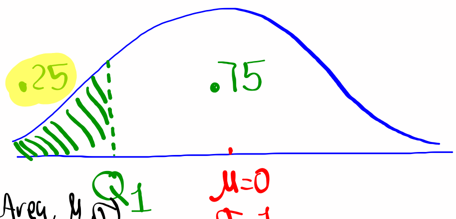
$= \boxed{.079}$

Find $Z = Q_1$

25% below — 75% above

Reverse Process

$\boxed{2nd}$ $\boxed{VARS}$

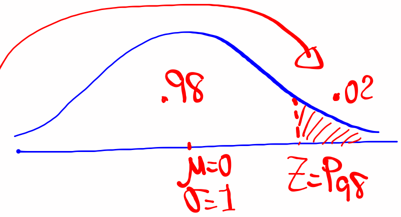


$Z = Q_1 =$ invNorm(Left Area, μ, σ)

$=$ invNorm(.25, 0, 1) $= \boxed{-.674}$

Find $Z = P_{98}$

98% below — 2% above



$Z = P_{98} =$ invNorm(.98, 0, 1) $= \boxed{2.054}$

Normal Prob. dist.:

1) Use x, $P(x = c) = 0$

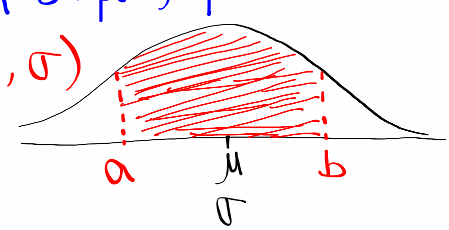
2) Dist. is Symmetric, bell-Shape, with total area $= 1$

3) Mean = Mode = Median

4) μ & σ are given in the problem.

$P(a < x < b)$ is the Corresponding area within the bell-Shape graph

= normalcdf(L, U, μ, σ)



$N(\mu, \sigma)$

Given $N(78, 10)$

Normal Prob. dist.

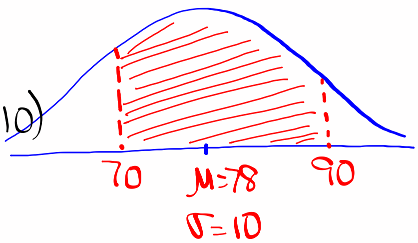
$\mu = 78$

$\sigma = 10$

$P(70 < x < 90)$

$= \text{normalcdf}(70, 90, 78, 10)$

$= \boxed{.673}$



Find $x = Q_3$, Round to a whole #

75% below

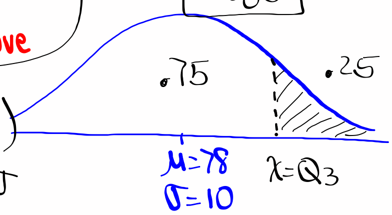
25% above

$\boxed{x \approx 85}$

$x = \text{invNorm}(.75, 78, 10)$

left Area μ σ

$= 84.745$



Consider a normal prob. dist. with the mean of 125 and standard deviation of 15.

$\mu = 125$

$\sigma = 15$

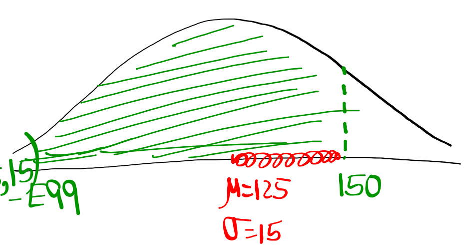
$N(125, 15)$

$P(x > 100)$

$= \text{normalcdf}(100, E99, 125, 15)$

$= \boxed{.952}$

$P(x < 150)$



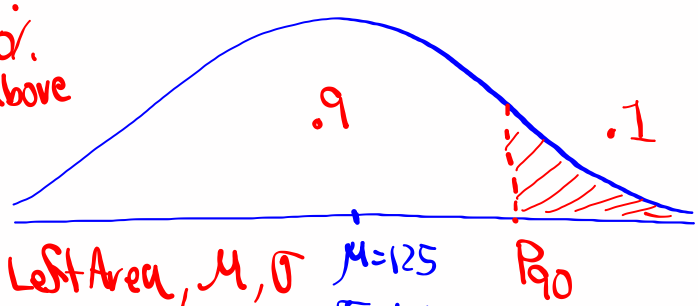
$= \text{normalcdf}(-E99, 150, 125, 15)$

$= \boxed{.952}$

Find $x = P_{90}$, Round to a whole #

90% below

10% above



Left Area, μ, σ

$x = P_{90} = \text{invNorm}(.9, 125, 15)$

$= 144.223 \approx \boxed{144}$

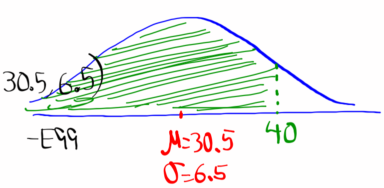
Ages of students are normally dist. with $\mu = 30.5$ Yrs and $\sigma = 6.5$ Yrs. $N(30.5, 6.5)$

If we randomly select $\boxed{\text{one student}}$, find the prob. that $\boxed{\text{his/her age}}$ is below 40.

x

$P(x < 40)$

$= \text{normalcdf}(-E99, 40, 30.5, 6.5)$



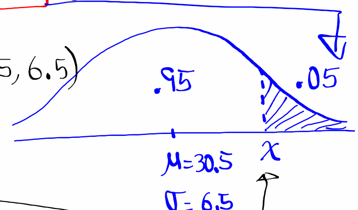
$\approx \boxed{.928}$

Find the age, round to 1-decimal, that separates the $\boxed{\text{top } 5\%}$ from the rest.

$x = \text{invNorm}(.95, 30.5, 6.5)$

$= 41.192$

$\approx \boxed{41.2}$



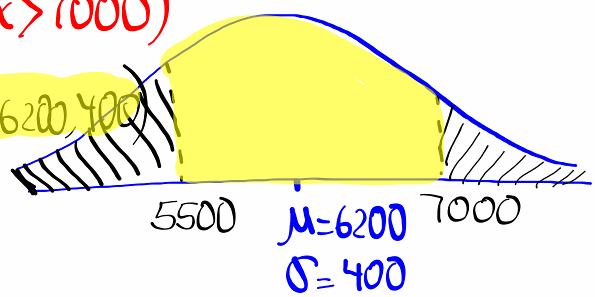
Salaries of all nurses are normally dist. with $\mu = 6200$ & $\sigma = 400$. $N(6200, 400)$

Find the prob. that one randomly selected (x) nurse makes below \$5500 or above \$7000.

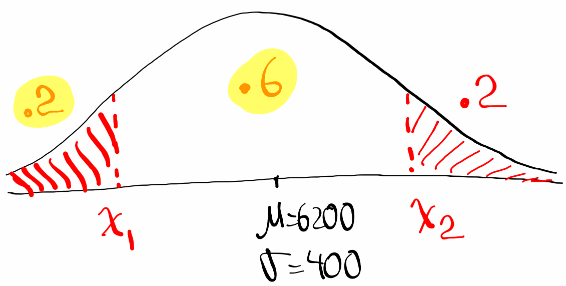
$P(x < 5500 \text{ OR } x > 7000)$

$= 1 - \text{normalcdf}(5500, 7000, 6200, 400)$

$= \boxed{.063}$



Find two salaries, round to whole #, that separate the middle 60% from the rest.



$x_1 = \text{invNorm}(.2, 6200, 400) \approx \boxed{5863}$

$x_2 = \text{invNorm}(.8, 6200, 400) \approx 6537$

SG 18 & SG 19

Clear all lists

Reset all lists

Store 2, 6, 10, 14 in L1

Find

$\boxed{\mu = 8}$ $\sigma = 4.472$ $\boxed{\sigma^2(\text{exact}) = 20}$

Take all Samples with $\boxed{\text{Size } 2}$ with replacement From this data.

2,2	2,6	2,10	2,14
6,2	6,6	6,10	6,14
10,2	10,6	10,10	10,14
14,2	14,6	14,10	14,14

Now Find $\bar{x}$ for each Sample

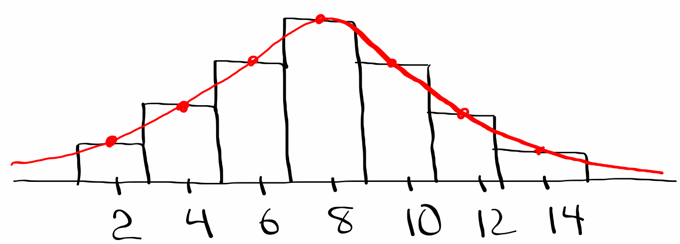
2	4	6	8
4	6	8	10
6	8	10	12
8	10	12	14

16 Means

$\bar{x}$	$P(\bar{x})$
2	1/16
4	2/16
6	3/16
8	4/16
10	3/16
12	2/16
14	1/16

$\bar{x}$	$P(\bar{x})$
2	1/16
4	2/16
6	3/16
8	4/16
10	3/16
12	2/16
14	1/16

Prob. dist. histogram



$\bar{x} \to L2$, $P(\bar{x}) \to L3$

Use L2 & L3 to find

$\boxed{\mu = 8}$ $\sigma = 3.162$ $\boxed{\sigma^2(\text{exact}) = 10}$

Clear all lists

Store 1, 3, 5 in L1

Use L1 to find

$\boxed{\mu = 3}$ $\sigma = 1.633$ $\boxed{\sigma^2(\text{exact}) = \frac{8}{3}}$

Take all Samples of $\boxed{\text{Size } 2}$ with replacement from this data.

1,1 1,3 1,5

3,1 3,3 3,5

5,1 5,3 5,5

Now find $\bar{x}$ of each Sample

1 2 3

2 3 4

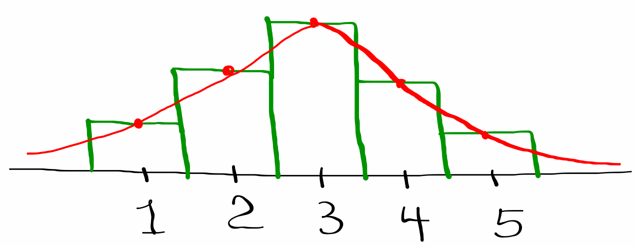
3 4 5

9 means

$\bar{x}$	$P(\bar{x})$
1	1/9
2	2/9
3	3/9
4	2/9
5	1/9

$\bar{x}$	$P(\bar{x})$
1	1/9
2	2/9
3	3/9
4	2/9
5	1/9

Draw Prob. dist. Histogram



$\bar{x} \to L2$, $P(\bar{x}) \to L3$

Use L2 & L3 to find

$\frac{\frac{8}{3}}{2} = \frac{4}{3}$

$\boxed{\mu = 3}$ $\sigma = 1.555$ $\boxed{\sigma^2(\text{exact}) = \frac{4}{3}}$

Central - Limit Theorem

$$\boxed{\mu_{\bar{x}} = \mu}$$

$$\boxed{\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}}$$

$$\boxed{\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}}$$

SG 20 (first 3 pages)

Class QZ 10

Consider a Poisson prob. dist. with $\mu = 8$.

1) find $P(x=10)$ = poissonpdf(8,10) = $\boxed{.099}$

2) find $P(x<15) = P(x \leq 14)$ = poissoncdf(8,14)

= $\boxed{.983}$